2-7 Videos Guide

2-7a

- The Midpoint Rule:
  - $\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + \cdots f(\overline{x_n})]$, where $\Delta x = \frac{b-a}{n}$ and $\overline{x_i} = \frac{x_{i-1}+x_i}{2}$, the midpoint of $[x_{i-1}, x_i]$
  - Error is $|E_M| \leq \frac{K(b-a)^3}{24n^2}$, where $|f''(x)| \leq K$ for $x \in [a, b]$

2-7b

- The Trapezoid Rule:
  - $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots 2f(x_{n-1}) + f(x_n)]$, where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$
  - Error is $|E_T| \leq \frac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ for $x \in [a, b]$

Exercises:

2-7c

- Use (a) the Trapezoidal Rule and (b) the Midpoint Rule to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)

  $\int_0^4 \sqrt{y}\cos y\ dy, \qquad n = 8$

2-7d

- a) Estimate the errors for T_{10} and M_{10} for $\int_1^2 e^{1/x}\,dx$.

  b) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) are accurate to within 0.0001?

2-7e

- Simpson's Rule:
  - $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$, where n is even, $\Delta x = \frac{b-a}{n}$, and $x_i = a + i\Delta x$
  - Error is $|E_S| \leq \frac{K(b-a)^5}{180n^4}$, where $\left|f^{(4)}(x)\right| \leq K$ for $x \in [a, b]$